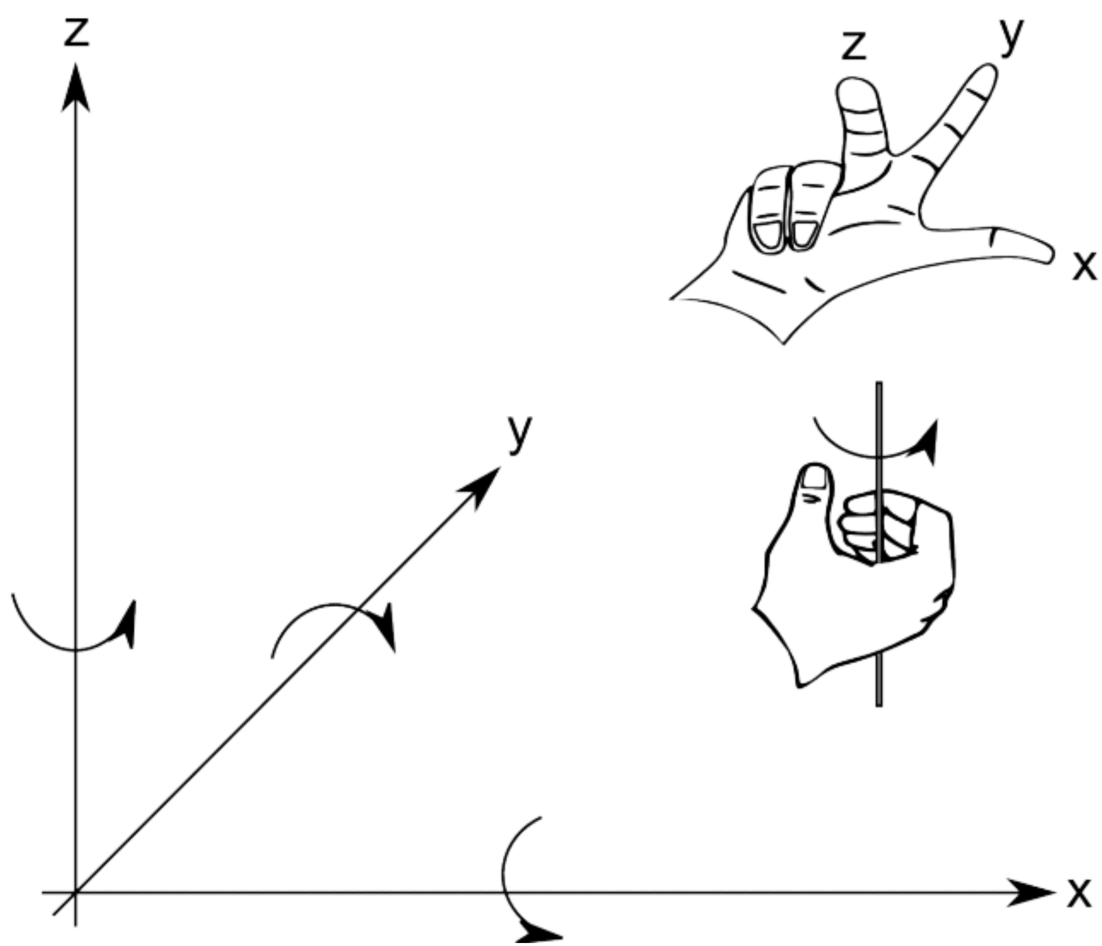
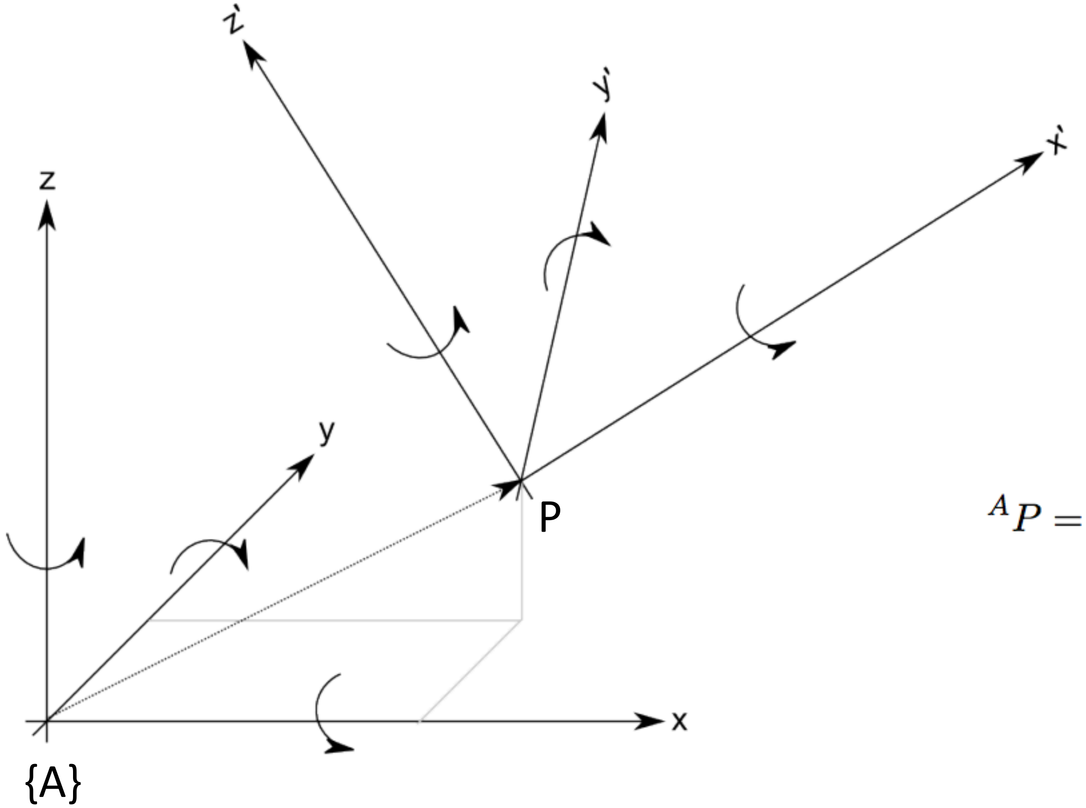
Kinematics I

Chapter 3

Coordinate Systems and Right-Hand Rule

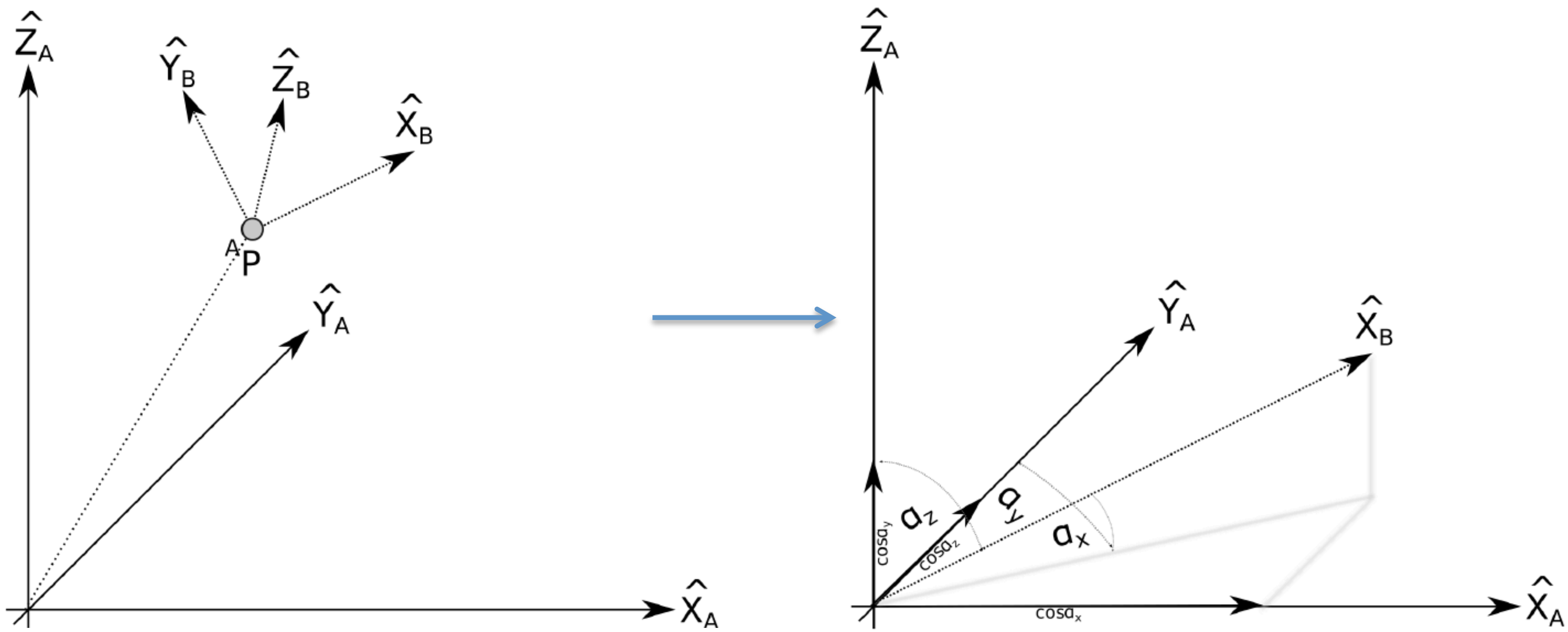


Nested Coordinate Systems



$$^{A}P = p_x \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + p_y \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + p_z \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Expressing Rotations



$$^{A}\hat{X}_B = (\hat{X}_B \cdot \hat{X}_A, \hat{X}_B \cdot \hat{Y}_A, \hat{X}_B \cdot \hat{Z}_A)^T$$

“Express X_B in the coordinate frame A”

Transformation Arithmetic

Rotation Matrix

$$^{A}P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}$$

$$^{A}Q = ^{A}_{B} R ^{B}Q + ^{A} P$$

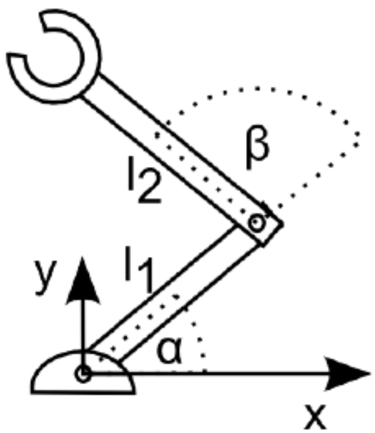
Euler Angles

$$^{A}_{B}R_{XYZ}(\gamma, \beta, \alpha) = \begin{bmatrix} cos\alpha & -sin\alpha & 0 \\ sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\beta & 0 & sin\beta \\ 0 & 1 & 0 \\ -sin\beta & 0 & cos\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\gamma & -sin\gamma \\ 0 & sin\gamma & cos\gamma \end{bmatrix}$$

Homogeneous Transform

$$\begin{bmatrix} ^{A}Q \end{bmatrix} = \left[\begin{array}{ccc|c} & ^{A}_{B}R & & ^{A}P \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} ^{B}Q \\ 1 \end{bmatrix}$$

Forward Kinematics (Arm)



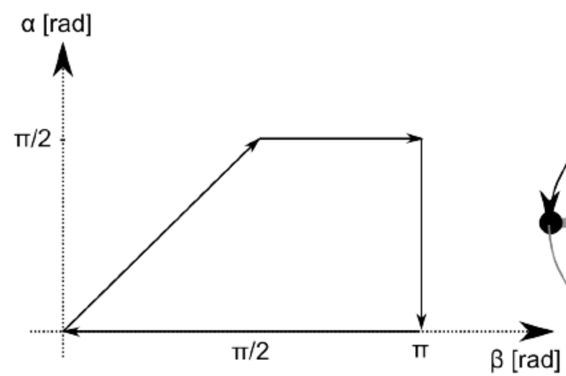
$$
\begin{aligned}
x_1 &= \cos\alpha l_1 \\
y_1 &= \sin\alpha l_1
\end{aligned}
$$

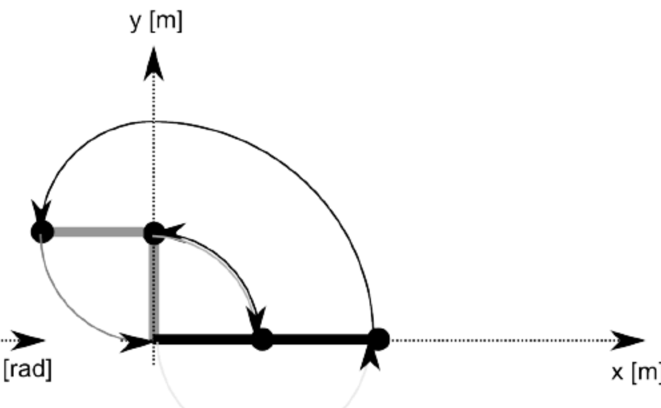
$$
\begin{aligned}
x_2 &= \cos(\alpha+\beta)l_2 + x_1 \\
y_2 &= \sin(\alpha+\beta)l_2 + z_1
\end{aligned}
$$

$$
\begin{aligned}
x &= \cos(\alpha+\beta)l_2 + \cos\alpha l_1 \\
y &= \sin(\alpha+\beta)l_2 + \sin\alpha l_1
\end{aligned}
$$

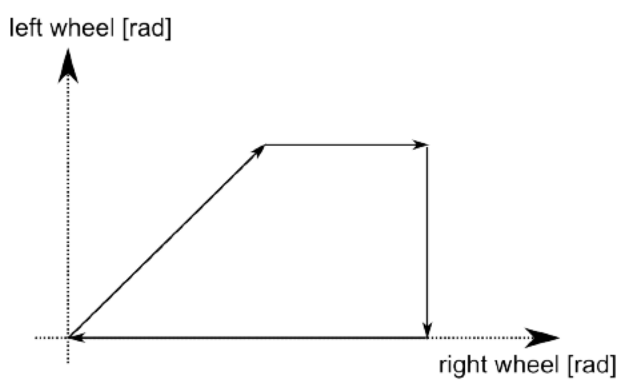
Holonomic vs. Non-Holonomic

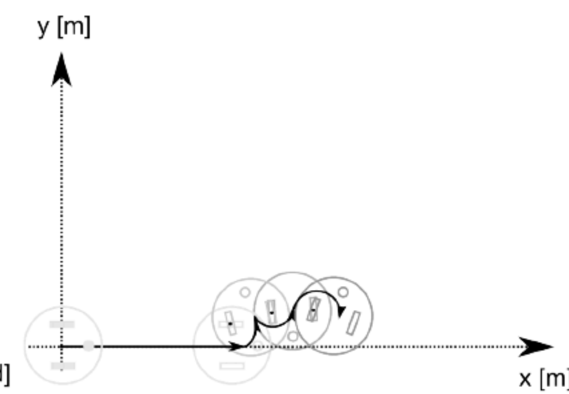
Manipulator





Diff. Wheels

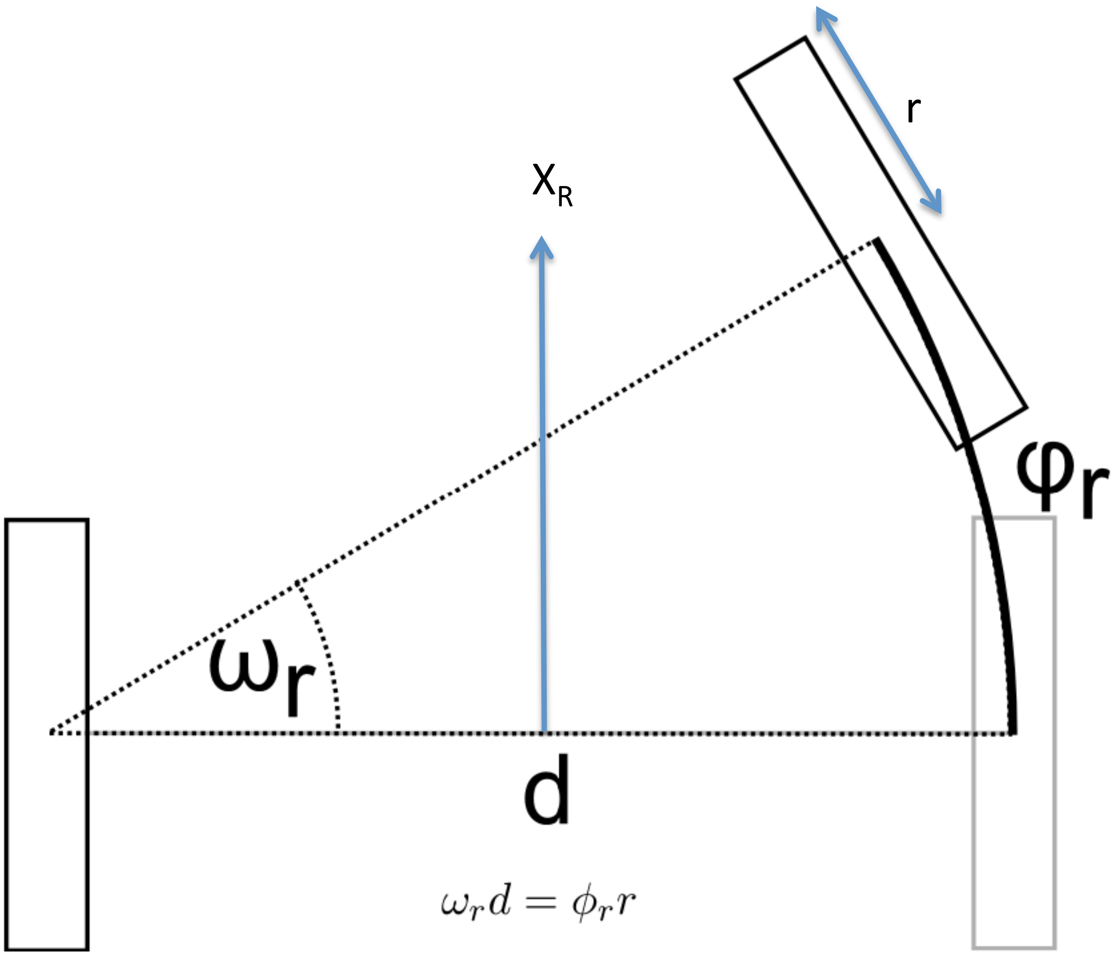




Configuration Space

Workspace

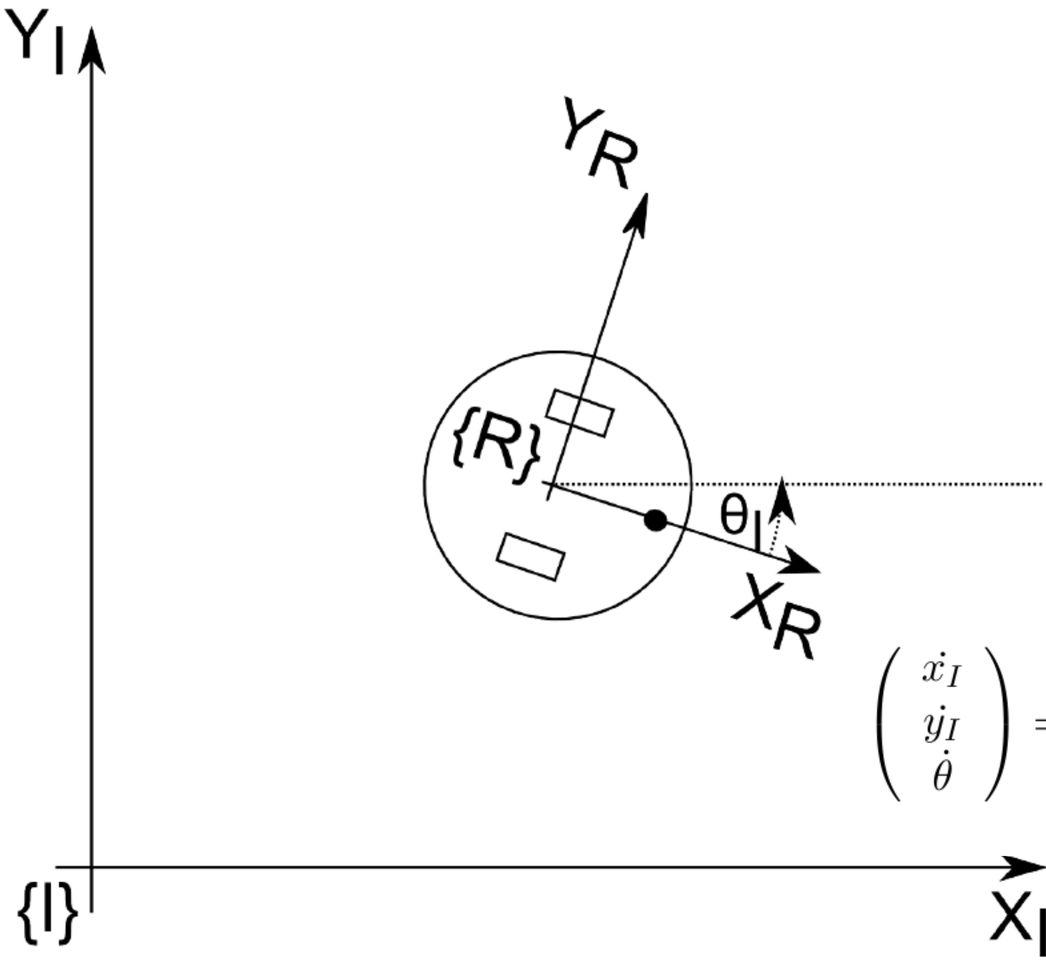
Kinematics of a Mobile Robot



$$\dot{x}_R = \frac{r\dot{\phi}_l}{2} + \frac{r\dot{\phi}_r}{2}$$

$$\omega_r d = \phi_r r$$

Kinematics of Mobile Robot



$$\dot{x}_{I,x} = cos(\theta)\dot{x}_R$$

$$\dot{x}_{I,y} = -sin(\theta)\dot{y}_R$$

$$\dot{x}_I = cos(\theta)\dot{x}_R - sin(\theta)\dot{y}_R$$

$$\dot{y}_I = sin(\theta)\dot{x}_R + cos(\theta)\dot{y}_R$$

$$\dot{\theta}_I = \dot{\theta}_R$$

$$\begin{pmatrix} \dot{x}_I \\ \dot{y}_I \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{r\dot{\phi}_l}{2} + \frac{r\dot{\phi}_r}{2} \\ 0 \\ \frac{\dot{\phi}_r r}{d} - \frac{\dot{\phi}_l r}{d} \end{pmatrix}$$