Localization: Part II

Chapter 9
Last Week

- Bayes’ rule provides a formal framework to use information about known features in a map.
- Together with “error propagation” this is known as “Markov Localization”.
- The problem can be computationally simplified using a “Particle Filter”.

\[ P(A|B) = \frac{P(A)P(B|A)}{P(B)} \]
Lab

- Calculate a position estimate (X,Y) from two range measurements
- Calculate the variance of the range measurement
- Calculate the variance of X,Y
- New: *Two* estimates for the robots location
  - Odometry
  - Triangulation
Brainstorming: How to fuse two sources of information for the same random variable?

• Problem statement
  – Given a prior distribution for the robot’s location
  – The range measurement (Gaussian distributed) from a known beacon
  – Required: Posterior distribution given the observation

• How to do this using the Markov localization example?
• How to do this using the Particle filter example?
Possible ways to merge information

- *Markov localization*: multiply a circular Gaussian distribution around the known beacon with the prior pose
- *Particle Filter*: Calculate the probability for every particle to obtain such a range measurement
- Or: calculating a new distribution based on the individual variances
Optimal fusion of two random variables

\[ \hat{q}_1 \quad \hat{q}_2 \]

\[ \sigma_1^2 \quad \sigma_2^2 \]

\[
\min_q S = \sum_{i=1}^{n} \frac{1}{\sigma_i} (q - \hat{q}_i)^2
\]

\[
q = \hat{q}_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} (\hat{q}_2 - \hat{q}_1)
\]

\[
\sigma^2 = \frac{1}{\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}}
\]
Example
Optimal fusion of two random variables

• Weighing each observation with its variance leads to an optimal estimate.
• The new variance is \textit{smaller} than either measurement’s variance!
• Adding information always helps.
• Careful: only works for independent random variables.
The Kalman Filter

• Other interpretation:
  – $q_1$ current value
  – $q_2$ is a prediction

\[ q = \hat{q}_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} (\hat{q}_2 - \hat{q}_1) \]

• Known as the perception update of the filter (action update as before)

• New estimate is a weighted sum between own estimate $q_1$ and prediction $q_2$

• $q_2 - q_1$ is known as \textit{Innovation} and $\frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}$ as the \textit{Kalman gain}
So far in this class...

- How a robot moves (kinematics) and how its uncertainty propagates
- How sensors work and how to extract information from them
- How to fuse information from different sources (to obtain a robot’s location)
- How to plan a robot’s motion
- Tons of useful stuff: trigonometry, statistics, RANSAC, least-squares, particle filter, electronics, ...
- Missing: treating maps / beacons as being also probabilistic, known as Simultaneous Localization and Mapping
Reminder of this class (4 weeks)

• Project
  – Model a business application with Sparki (coverage, delivery, search and rescue etc.)
  – Next Monday: Design review
  – Explain what your problem is and how you will solve it
  – **Need to use concepts from class**
  – Final deliverable: presentation, demo and 1-minute video

• Debates
  – Oxford style: Pro, Contra, Synthesis
  – **Need to be anchored in concepts from class**
  – **Need to perform literature review (magazine articles, policy articles, technical publications)**
Summary

• Multiple ways to fuse different information for a common random variable
• Markov localization, particle filter, Kalman filter
• The more information, the better