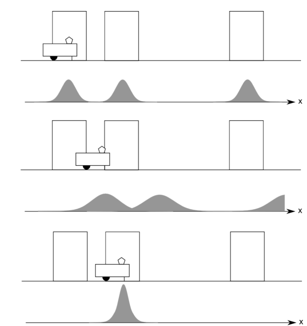
Localization: Part II

Chapter 9

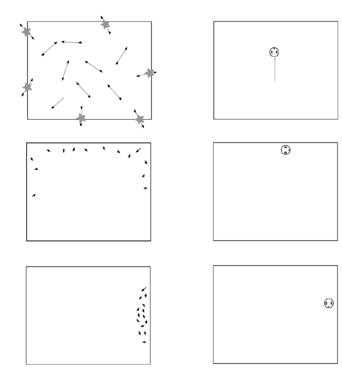
Last Week

- Bayes' rule provides a formal framework to use information about known features in a map

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)}$$

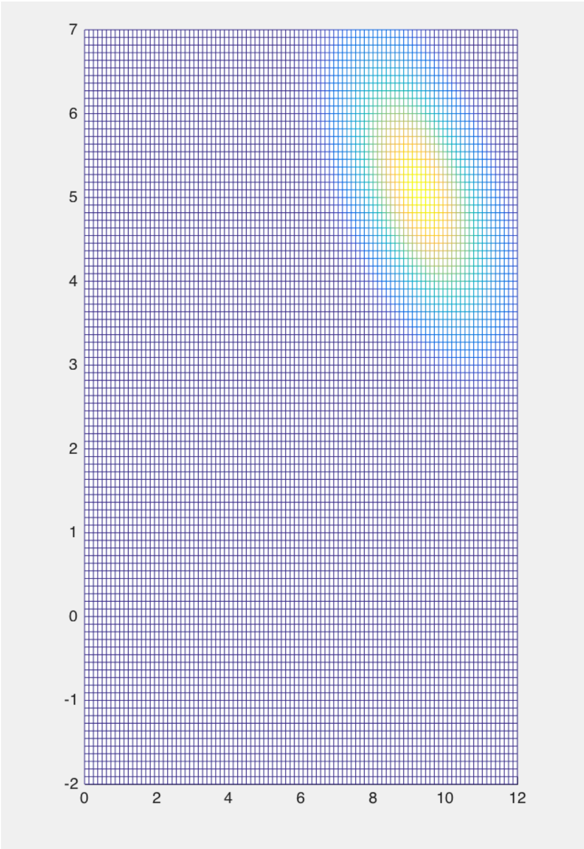


- Together with "error propagation" this is known as "Markov Localization"
- The problem can be computationally simplified using a "Particle Filter"



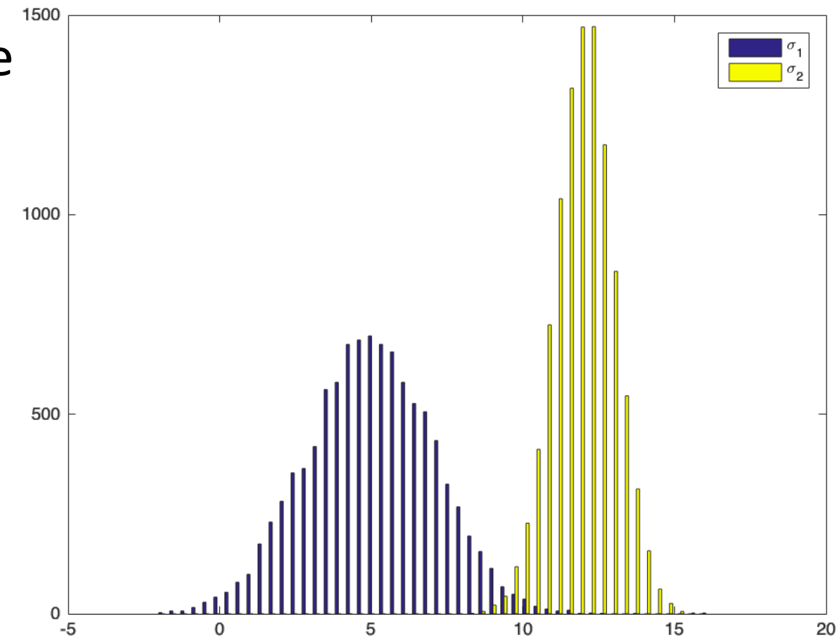
Lab

- Calculate a position estimate (X,Y) from two range measurements
- Calculate the variance of the range measurement
- Calculate the variance of X,Y
- New: *Two* estimates for the robots location
  - Odometry
  - Triangulation



Brainstorming: How to fuse two sources of information for the same random variable?

- Problem statement
  - Given a prior distribution for the robot's location
  - The range measurement (Gaussian distributed) from a known beacon
  - Required: Posterior distribution given the observation
- How to do this using the Markov localization example?
- How to do this using the Particle filter example?



Possible ways to merge information

- *Markov localization*: multiply a circular Gaussian distribution around the known beacon with the prior pose
- *Particle Filter*: Calculate the probability for every particle to obtain such a range measurement
- Or: calculating a new distribution based on the individual variances

Optimal fusion of two random variables

$$\hat{q}_1 \qquad \hat{q}_2$$

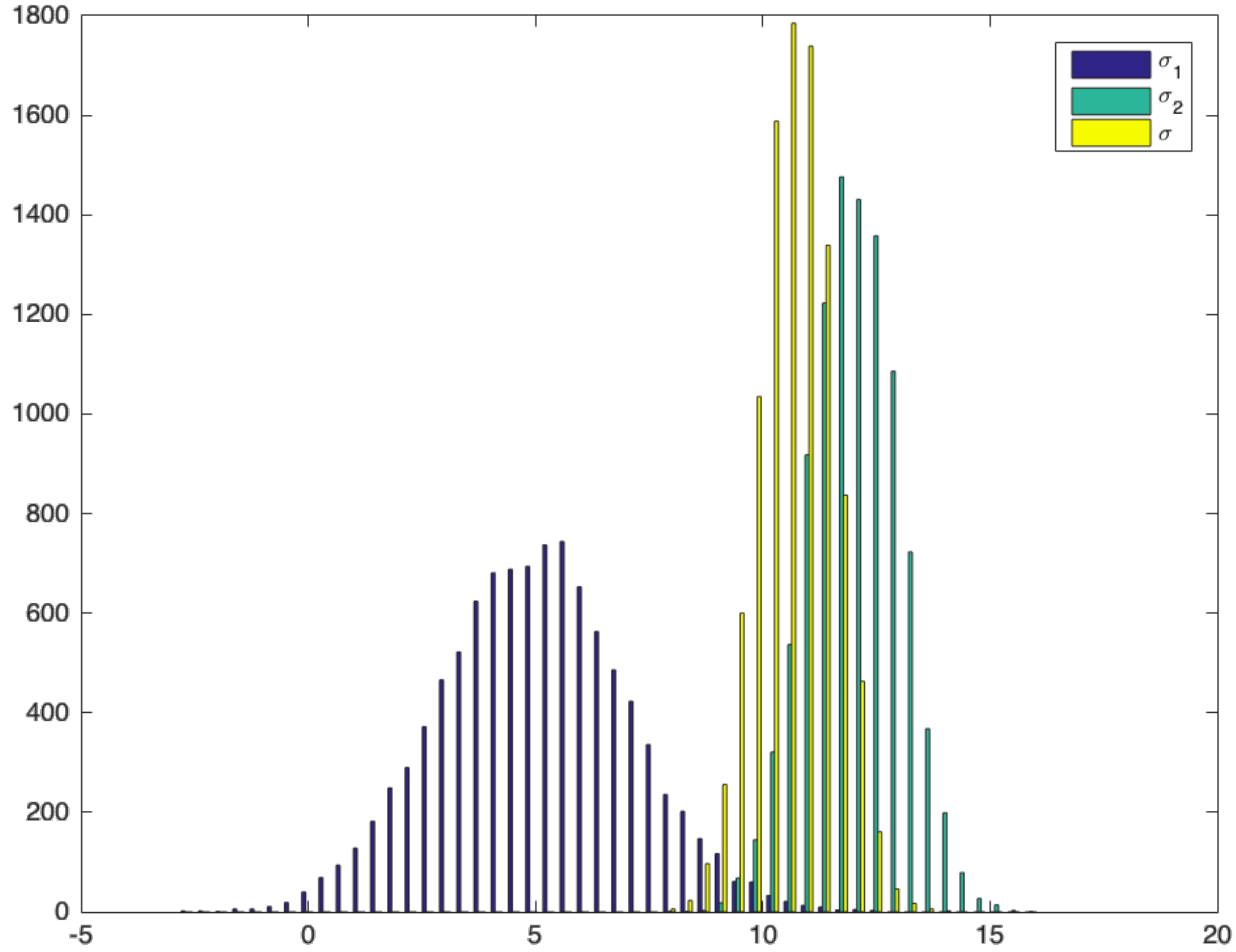
$$\sigma_1^2 \qquad \sigma_2^2$$

$$\min_q S = \sum_{i=1}^{n} \frac{1}{\sigma_i}(q - \hat{q}_i)^2$$

↓

$$q = \hat{q}_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}(\hat{q}_2 - \hat{q}_1) \qquad \sigma^2 = \frac{1}{\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}}$$

Example



1800
1600
1400
1200
1000
800
600
400
200
0
-5
0
5
10
15
20
σ_1
σ_2
σ

Optimal fusion of two random variables

- Weighing each observation with its variance leads to an optimal estimate
- The new variance is *smaller* than either measurement's variance!
- Adding information *always* helps
- Careful: only works for independent random variables

The Kalman Filter

- Other interpretation:
  - q_1 current value
  - q_2 is a prediction

$$q = \hat{q}_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}(\hat{q}_2 - \hat{q}_1)$$

- Known as the perception update of the filter (action update as before)
- New estimate is a weighted sum between own estimate q_1 and prediction q_2
- q_2-q_1 is known as *Innovation* and $\frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}$ as the *Kalman gain*

So far in this class…

- How a robot moves (kinematics) and how its uncertainty propagates
- How sensors work and how to extract information from them
- How to fuse information from different sources (to obtain a robot's location)
- How to plan a robot's motion
- Tons of useful stuff: trigonometry, statistics, RANSAC, least-squares, particle filter, electronics, …
- Missing: treating maps / beacons as being also probabilistic, known as *Simultaneous Localization and Mapping*

Reminder of this class (4 weeks)

- Project
  - Model a business application with Sparki (coverage, delivery, search and rescue etc.)
  - Next Monday: Design review
  - Explain what your problem is and how you will solve it
  - **Need to use concepts from class**
  - Final deliverable: presentation, demo and 1-minute video
- Debates
  - Oxford style: Pro, Contra, Synthesis
  - **Need to be anchored in concepts from class**
  - **Need to perform literature review (magazine articles, policy articles, *technical publications*)**

Summary

- Multiple ways to fuse different information for a common random variable
- Markov localization, particle filter, Kalman filter
- The more information, the better