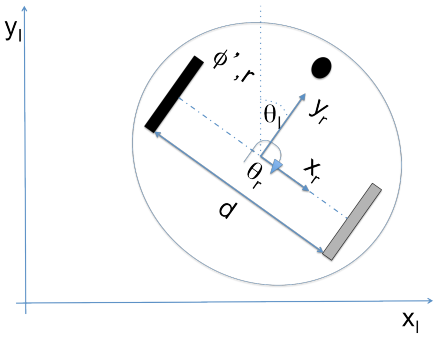
CS3202 Introduction to Robotics
Homework 1

Please provide all answers and its derivations (if applicable) on this sheet and its back. Other hand-ins (scratch paper) will not be considered.

1. Kinematics: Forward Kinematics

Your robotic vacuum cleaner is broken, but you have not given up on it! Unfortunately, only its left motor is still working. Assume this wheel drives with speed $\dot{\varphi}$ and radius r. Calculate the *speed* of the robot in the *robot coordinate frame* (X_r, Y_r, θ_r) that is coincidental with the robot's center and assume that the axis diameter is d, the wheels are $d/2$ away from the robots' center.

a. Write down expressions for the *speed* in the frame (X_r, Yr, θ_r) given $\dot{\varphi}$ and r assuming that the broken wheel does not spin.



b. Write down an expression for the speed in the inertial frame (X_I, Y_I, θ_I) as a function of $\dot{\varphi}$ and r. Infer the orientation of θ_I from the drawing.

2. Inverse Kinematics

Consider now a differential wheels robot with the following forward kinematics:

$$\begin{pmatrix} \dot{x_R} \\ \dot{y_R} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \frac{r\dot{\phi_l}}{2} + \frac{r\dot{\phi_r}}{2} \\ 0 \\ \frac{\dot{\phi_r} r}{d} - \frac{\dot{\phi_l} r}{d} \end{pmatrix}$$

Derive an expression for $\dot{\varphi}_l$ and $\dot{\varphi}_r$ as a function of the desired speed $\dot{x}_R$ and $\dot{\theta}$.